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Position Paper

An Archimedean obstruction and ordinal sums for t-norms on the norm-first multi-fuzzy cube

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ABSTRACT

A multi-fuzzy set assigns to each object a k-tuple of membership degrees, so a k-tuple valued semantics needs an order on the cube I^k and a conjunction compatible with it. Ordering I^k by the norm-first lexicographic order $\preccurlyeq$ gives a bounded chain C_k, and the admissible conjunctions are the t-norms on that chain. Throughout, continuity of a t-norm is with respect to the order topology induced by $\preccurlyeq$. Two facts are proved here. First, for $k \geq 2$ the chain C_k carries no continuous Archimedean t-norm. The proof is self-contained: every linear continuum carrying a continuous Archimedean t-norm is order-separable, and C_k has cellularity $\mathfrak{c}$ for $k \geq 2$. So the Łukasiewicz and the product operation cannot be transported to C_k itself, and a scale is needed for them. Second, non-idempotent conjunction as such does not need a scale. The closed intervals of C_k that are order-isomorphic to $([0,1], \leq)$ are characterised as exactly the arcs along which the norm and the first $k-2$ coordinates are constant, and these arcs serve as ordinal-sum summands. The resulting operations are continuous, divisible and residuated, so each of them makes C_k a BL-chain, and a single non-idempotent summand already makes it one that is not a Gödel chain. There are $2^{\mathfrak{c}}$ of them, against only $\mathfrak{c}$ continuous t-norms on the unit interval. A representation of every continuous t-norm on C_k as such an ordinal sum is not claimed.

1. Introduction

Fuzzy sets [1] record membership by a single number, and most of their extensions replace that number by a richer object: a pair of degrees [2], a set of degrees [3], or a fuzzy set of degrees [4]. Multi-fuzzy sets were introduced by Sebastian and Ramakrishnan [5] as maps whose values are k-tuples of membership degrees, so that several criteria are carried side by side instead of being aggregated in advance; the topology, the correspondence with other generalisations of fuzzy sets and several applications have been developed since [6–10]. Once membership degrees are tuples, the truth-value set is the cube I^k, where $I = [0,1]$, and the coordinatewise order makes it a lattice but not a chain. Every L-valued semantics in the sense of Goguen [11] is available on that lattice, and t-norms on product lattices and on bounded lattices have been studied in detail [12–14]. A chain, however, is what the standard apparatus of t-norm based logic asks for [15–18], since prelinearity, ordinal sums and the completeness machinery are all read on linearly ordered structures [19,20].

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